$\therefore\ P=\frac{8}{9}F$.

Assuming \$100 for the face of the note, the proceeds will be \$$88\frac{8}{9}$, and the discount or interest \$$11\frac{1}{9}$.

From (2), $\text{Time}=\dfrac{\text{Int.}}{P\times r}=\dfrac{11\frac{1}{9}}{88\frac{8}{9}\times\frac{9}{40}}=\frac{5}{9}$ year=200 days.

Also solved by *G. B. M. ZERR*, and *H. C. WHITAKER*.

ALGEBRA.

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119. Proposed by HARRY S. VANDIVER, Bala, Montgomery County, Pa.

Given $\tan x=x+\dfrac{x^3}{3}+\dfrac{2x^5}{3\times5}+\dfrac{17x^7}{3^2\times5\times7}+\dfrac{62x^9}{3^2\times5\times7\times9}\ldots\ldots$

Find the general term and interval of convergence of this series.

I. Solution by COOPER D. SCHMITT, M. A., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Using the notation of the Calculus, let $\tan x$ be represented by $f(x)$ or simply f. Then dy/dx will be $f'(x)$ or more simply f; similarly, f'', f''', f^{iv}, f^{v}, etc., will be used. We have then by differentiating:

$$f(x)=\tan x \qquad f(0)=0.$$

$$f'=\sec^2 x \qquad f'(0)=1.$$

$$f''=2\sec^2 x\tan x=2ff' \qquad f''(0)=0.$$

$$f'''=2ff''+2f'^2 \qquad f'''(0)=2.$$

$$f^{\text{iv}}=6f'f''+2ff''' \qquad f^{\text{iv}}(0)=0.$$

$$f^{\text{v}}=6f''^2+8f'f'''+2ff^{\text{iv}} \qquad f^{\text{v}}(0)=16.$$

$$f^{\text{vi}}=20f''f'''+10f'f^{\text{iv}}+2ff^{\text{v}} \qquad f^{\text{vi}}(0)=0.$$

$$f^{\text{vii}}=20f''+30f''f^{\text{iv}}+12f'f^{\text{v}}+2ff^{\text{vi}} \qquad f^{\text{vii}}(0)=272.$$

We see that the *even* differential coefficients vanish and the *odd* follow a remarkable law: The first term is the middle term in the expansion by the Binomial Theorem and the following terms are the double of the successive coefficients of the Binomial Theorem.

By this law we can write any of the odd derivatives, thus:

$$f^{\text{ix}}=70f^{\text{iv}2}+112f'''f^{\text{v}}+56f''f^{\text{vi}}+16f'f^{\text{vii}}+2ff^{\text{viii}} \qquad f^{\text{ix}}(0)=7936.$$

$$f^{\text{xi}}=252f^{\text{v}2}+420f^{\text{iv}}f^{\text{vi}}+240f'''f^{\text{vii}}+90f''f^{\text{viii}}+20f'f+2^{\text{ix}}ff^{\text{x}} \qquad f^{\text{xi}}(0)=353792.$$

$$f^{\text{xiii}}=924f^{\text{vi}2}+1584f^{\text{v}}f^{\text{vii}}+990f^{\text{iv}}f^{\text{viii}}+440f'''f^{\text{ix}}+132f''f^{\text{x}}+24f'f^{\text{xi}}+2ff^{\text{xii}}$$

whence $f^{\text{xiii}}(0)=22368256$, and thus indefinitely.

That is, in writing f^{xiii}, I write the coefficients of $(a+b)^{12}$, which are 1, 12, 66, 220, 495, 792, 924, etc. I set down 924 and double each of the others and get the result as given above.

Substituting the values obtained in MacLaurin's Formula, we have :

$$\tan x=x+\frac{2x^3}{3\,!}+\frac{16x^5}{5\,!}+\frac{272x^7}{7\,!}+\frac{7936x^9}{9\,!}+\frac{353792x^{11}}{11\,!}+\frac{22368256x^{13}}{13\,!}+\text{ etc.}$$

$$=x+\frac{x^3}{3}+\frac{2x^5}{3\times5}+\frac{17x^7}{3^2\times5\times7}+\frac{62x^9}{3^2\times5\times7\times9}+\frac{1382x^{11}}{3^2\times5^2\times7\times9\times11}$$

$$+\frac{21844x^{13}}{3^3\times5^2\times7\times9\times11\times13}.$$

II. Solution by H. C. WHITAKER, A. M., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa., and J. SCHEFFER, A. M., Hagerstown, Md.

The general term of the series is

$$\frac{2^{2n}(2^{2n}-1)}{2n\,!}Bx^{2n-1},$$

where B (Bernoulli) stands for the coefficient of x^n in the expansion of

$$\frac{x}{e^x-1}=\frac{x}{\frac{x}{1}+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\text{etc.}}$$

120. Proposed by JOSIAH H. DRUMMOND, LL. D., Portland, Me.

A hollow sphere has within it a solid sphere; a quantity of water equal to $1/m$ of the capacity of the hollow sphere is poured in and just covers the solid sphere. Prove that there are two solid spheres, either of which answers the conditions; also find the maximum value $1/m$, beyond which the question is not possible.

I. Solution by J. A. COLSON, Searsport, Me.

The volume of the inner sphere (radius r)+the volume of the water=the volume of a segment whose radius is R and whose height is $2r$; that is

$$\tfrac{4}{3}\pi r^3+\frac{4}{3m}\pi R^3=\tfrac{1}{3}\pi(2r)^2(3R-2r),$$

which reduces to $r^3-Rr^2+R^3/3m=0\ldots.(1)$.

Hence, by Cardan's Method one value of r is

$$r_1=\tfrac{1}{3}R+\sqrt[3]{\Big[\tfrac{1}{2}\Big[-\frac{R^3}{27}\Big(\frac{9}{m}-2\Big)+\frac{R^3}{9}\sqrt{\Big(\frac{9}{m^2}-\frac{4}{m}\Big)}\Big]\Big]}$$

$$+\sqrt[3]{\Big[\tfrac{1}{2}\Big[-\frac{R^3}{27}\Big(\frac{9}{m}-2\Big)-\frac{R^3}{9}\sqrt{\Big(\frac{9}{m^2}-\frac{4}{m}\Big)}\Big]\Big]}\ldots.(2).$$